

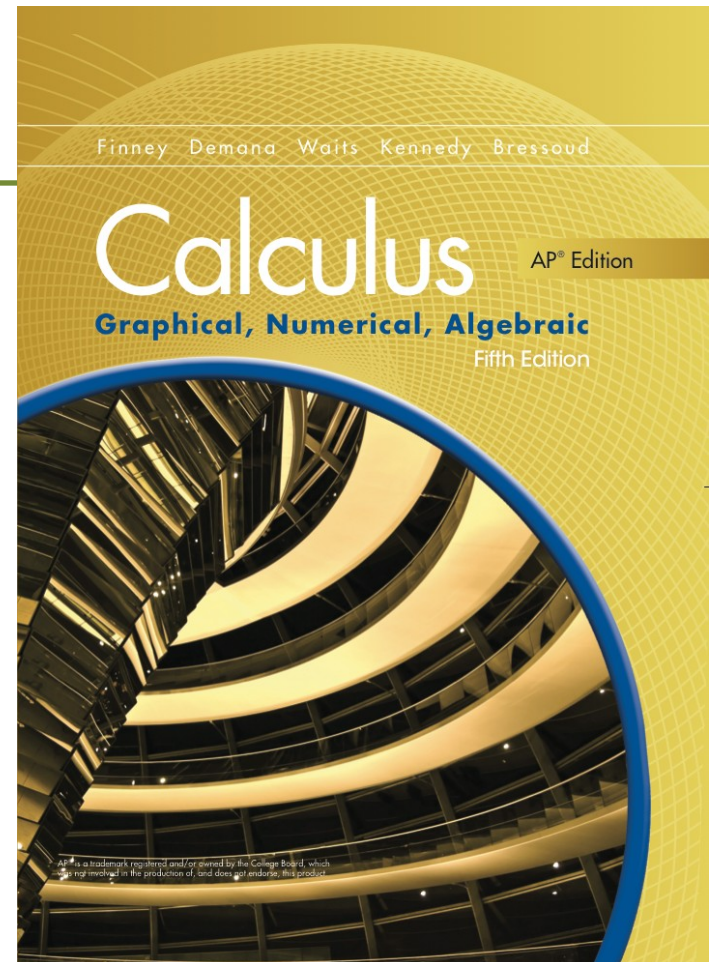
# Chapter 6

## Applications of Derivatives



### Section 6.3

## Definite Integrals and Antiderivatives



# Quick Review

Find  $dy/dx$

1.  $y = -\sin x$

2.  $y = \cos x$

3.  $y = \ln(\sec x)$

4.  $y = \ln(\cos x)$

5.  $y = x \ln x$

6.  $y = xe^x$

7.  $y = \tan^{-1} x$

# Quick Review Solutions

Find  $dy/dx$

1.  $y = -\sin x$      $dy/dx = -\cos x$

2.  $y = \cos x$      $dy/dx = -\sin x$

3.  $y = \ln(\sec x)$      $dy/dx = \tan x$

4.  $y = \ln(\cos x)$      $dy/dx = -\tan x$

5.  $y = x \ln x$      $dy/dx = 1 + \ln x$

6.  $y = xe^x$      $dy/dx = xe^x + e^x$

7.  $y = \tan^{-1} x$      $dy/dx = \frac{1}{x^2 + 1}$

# What you'll learn about

- Properties of definite integrals
- Average value of a function
- Mean value theorem for definite integrals

... and why

Working with the properties of definite integrals helps

us to understand better the definite integral.

Connecting derivatives and definite integrals sets the stage for the Fundamental Theorem of Calculus.

# Rules for Definite Integrals

- 1. Zero:**  $\int_a^a f(x) dx = 0$
- 2. Additivity:**  $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$
- 3. Order of Integration:**  $\int_b^a f(x) dx = - \int_a^b f(x) dx$
- 4. Constant Multiple:**  $\int_a^b kf(x) dx = k \int_a^b f(x) dx$  Any number  $k$   
 $\int_a^b -f(x) dx = - \int_a^b f(x) dx$   $k = -1$

# Rules for Definite Integrals

**5. Sum and Difference:**  $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

**6. Max-Min Inequality:** If  $\max f$  and  $\min f$  are the maximum and minimum values of  $f$  on  $[a, b]$ , then

$$\min f \cdot (b - a) \leq \int_a^b f(x) dx \leq \max f \cdot (b - a).$$

**7. Domination:**  $f(x) \geq g(x)$  on  $[a, b] \Rightarrow \int_a^b f(x) dx \geq \int_a^b g(x) dx$

$$f(x) \geq 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x) dx \geq 0 \quad g = 0$$

# Example Using the Rules for Definite Integrals

Suppose  $\int_1^1 f(x) dx = 5$ ,  $\int_1^4 f(x) dx = -2$ , and  $\int_1^1 h(x) dx = 7$ .

Find  $\int_4^1 f(x) dx$  if possible.

$$\int_4^1 f(x) dx = 2$$

# Example Using the Rules for Definite Integrals

Suppose  $\int_1^1 f(x) dx = 5$ ,  $\int_1^4 f(x) dx = -2$ , and  $\int_1^1 h(x) dx = 7$ .

Find  $\int_1^4 f(x) dx$  if possible.

$$\int_1^4 f(x) dx = 5 + (-2) = 3$$



# Example Using the Rules for Definite Integrals

Suppose  $\int_1^1 f(x) dx = 5$ ,  $\int_1^4 f(x) dx = -2$ , and  $\int_1^1 h(x) dx = 7$ .

Find  $\int_2^2 h(x) dx$  if possible.

Not enough information is given.

# Average (Mean) Value

If  $f$  is integrable on  $[a, b]$ , its average (mean) value on  $[a, b]$  is

$$\text{avg}(f) = \frac{1}{b-a} \int_a^b f(x) dx$$

# Example Applying the Definition

Find the average value of  $f(x) = 2 - x^2$  on  $[0, 4]$ .

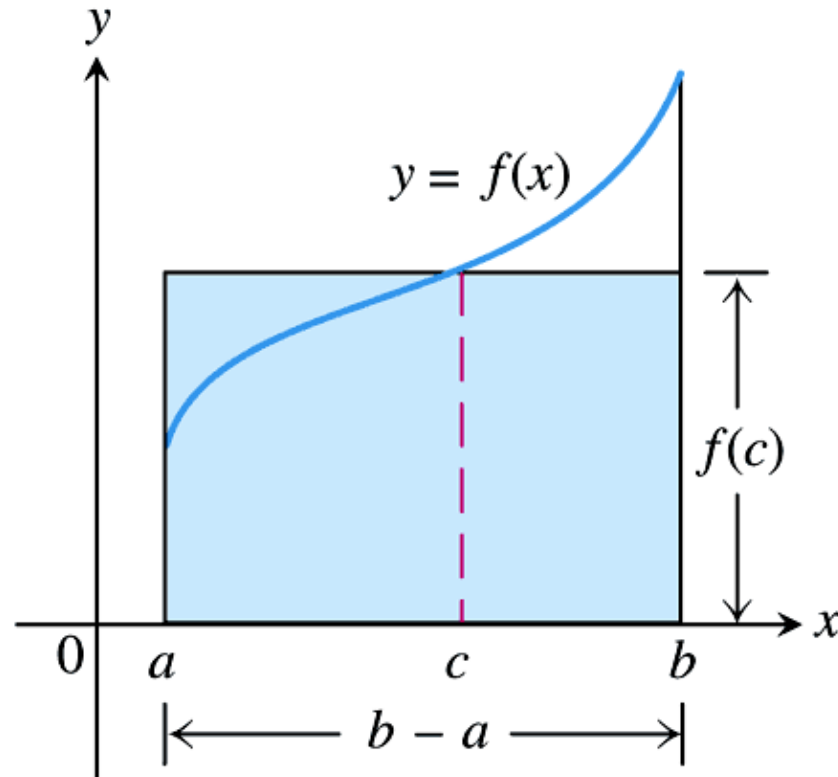
$$\begin{aligned} \text{avg}(f) &= \frac{1}{b-a} \int_a^b f(x) dx \\ &= \frac{1}{4-0} \int_0^4 (2 - x^2) dx \quad \text{Use NINT to evaluate the integral.} \\ &= \frac{1}{4} \left( 2x - \frac{x^3}{3} \right) \Big|_0^4 \\ &= \frac{10}{3} \end{aligned}$$

# The Mean Value Theorem for Definite Integrals

If  $f$  is continuous on  $[a, b]$ , then at some point  $c$  in  $[a, b]$ ,

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

# The Mean Value Theorem for Definite Integrals



# The Derivative of an Integral

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

# Quick Quiz Sections 6.1 – 6.3

You should solve the following problems without using a calculator.

1. If  $\int_a^b f(x) dx = a + 2b$ , then  $\int_a^b (f(x) + 3) dx =$

(A)  $a + 2b + 3$

(B)  $3b - 3a$

(C)  $4a - b$

(D)  $5b - 2a$

(E)  $5b - 3a$

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(E)  $5b - 3a$



# Quick Quiz Sections 6.1 – 6.3

2. The expression  $\frac{1}{20} \left( \sqrt{\frac{1}{20}} + \sqrt{\frac{2}{20}} + \sqrt{\frac{3}{20}} + \dots + \sqrt{\frac{20}{20}} \right)$

is a Riemann sum approximation for

(A)  $\int_0^1 \sqrt{\frac{x}{20}} dx$

(B)  $\int_0^1 \sqrt{x} dx$

(C)  $\frac{1}{20} \int_0^1 \sqrt{\frac{x}{20}} dx$

(D)  $\frac{1}{20} \int_0^1 \sqrt{x} dx$

(E)  $\frac{1}{20} \int_0^{20} \sqrt{x} dx$

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(E)  $\frac{1}{20} \int_0^{20} \sqrt{x} dx$

# Quick Quiz Sections 6.1 – 6.3

3. What are all values of  $k$  for which  $\int_2^k x^2 dx = 0$ ?

(A) - 2

(B) 0

(C) 2

(D) - 2 and 2

(E) - 2, 0, and 2

# Quick Quiz Sections 6.1 – 6.3

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(D) - 2 and 2

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